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ABSTRACT

Factor analysis attempts to study how different objects group together to form factors with the purposes of: (1) reducing the number of factorable entities (e.g., variables) with which the researcher needs to deal; (2) searching data for qualitative and quantitative differences; and (3) testing hypotheses (R. Gorsuch, 1983). While most factor analytic studies focus on finding factors to represent groups of variables, other factor analyses that look at factoring people and occasions offer many possibilities to researchers. These alternative forms of factor analysis are discussed, with an emphasis on Q-technique, which attempts to identify types of people, as it may be of particular interest to psychologists and educators. (Contains 3 tables and 18 references.) (Author/SLD)

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Factor Analysis of People Rather than Variables: Q and Other Two-Mode Factor Analytic Models

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Abstract

Factor analysis attempts to study how different objects group together to form factors, with the purposes of (a) reducing the number of factorable entities (e.g., variables) that the researcher needs to deal with, (b) searching data for qualitative and quantitative differences, and (c) testing hypotheses (Gorsuch, 1983). While most factor analytic studies focus on finding factors to represent groups of variables, other factor analyses look at factoring people and occasions offer many possibilities to researchers. The present paper discusses these alternative forms of factor analysis, with an emphasis on Q-technique, which attempts to identify types of people, as it may be of particular interest to psychologists and educators.

Factor analysis is a statistical procedure that identifies and summarizes relationships among factored entities (usually variables) as they are measured across many "replicates" (usually individuals). Traditional factor analysis, as defined by Gorsuch (1983), aims to "summarize the interrelationships among the variables in a concise but accurate manner" (p. 2). The procedure attempts to study how different objects group together to form factors, with the purposes of (a) reducing the number of factorable entities that the researcher needs to deal with, (b) searching data for qualitative and quantitative differences, and (c) testing hypotheses (Gorsuch, 1983). Kerlinger (1986) stated,

> Factor analysis serves the cause of scientific parsimony. It reduces the multiplicity of tests and measures to greater simplicity. It tells us, in effect, what tests or measures belong together... It thus reduces the number of variables with which the scientist must cope. (p. 569)

The theory behind factor analysis has been available to researchers since Spearman first proposed it at the beginning of the century, but actually conducting analyses has only become commonplace only with the advent of computers and software that can perform the complicated computations (Thompson & Dennings, 1993).

Factor analysis begins by using the raw data matrix to estimate a matrix of association. Although many different matrices are available for use (e.g., variance-covariance matrix), most exploratory factor analysis procedures utilize the traditional correlation, or Pearson r matrix (Carr, 1992). The matrix of association is computed from a raw data matrix where the columns represent the different items being factored and the rows contain scores from the items that are being replicated across the factored items. The most common form of factor analysis is the R-technique, which studies the relationships between variables (columns) as they are measured across individuals (rows). However, there are additional types of factor analysis that look at different relationships that offer useful information, despite being used less frequently.

The purpose of the current paper is to explore these alternative forms of two-mode factor analysis. Uses for different factor analysis techniques will be discussed. In addition, examples of the different techniques will be offered. Q-technique factor analysis will be examined in-depth and different considerations for the technique will be discussed.

History of Alternative Factor Analysis Models

In 1917 Sir Cyril Burt proposed that people, instead of variables, could be factored through factor analysis techniques (Campbell, 1996; Cattell, 1978). In 1935 Sir

Godfrey Thomson considered the possibility of using factor analysis to correlate people and named this procedure Q-technique (in contrast to the traditional R-technique)(Stephenson, 1953). However, Thomson was skeptical about the benefits of Q-technique and did not pursue the idea further. In the same year, and independently of Thomson, William Stephenson also considered the possibility of factoring people instead of variables in factor analysis procedures (Stephenson, 1953). Contrary to Thomson, however, Stephenson pursued the idea and debated the pros and cons of Q-technique factor analysis with Burt for the next several years (Stephenson, 1953).

The "Data Box"

Around World War II Raymond Cattell proposed the "data box" for use in conceptualizing factor analysis (Cattell, 1966). The data box is a theoretical three-dimensional box that is used to order information by the different ways that modes of information can be related. Cattell proposed that three features can be used to describe the data from any given study. It is these three dimensions that define the data box. In factor analysis these dimensions, called "modes," are individuals, variables, and occasions. While variables and subjects require little explanation, occasions can refer to different times of measurement, different methods of measurement, etc. For this reason, the "occasions" dimension is sometimes called "conditions" (Gorsuch, 1983).

Each mode defines one of three edges of the data box when the data box is oriented such that three surfaces of the box are simultaneously visible. Each surface of the data box is defined by the intersection of two of these dimensions. For example, one surface of the box is contained by the individuals and variables dimensions. This face of the data box signifies that variables and individuals are being considered in the study and the occasions mode is being held constant.

Alternative Forms of Two-Mode Factor Analysis

As mentioned previously, most factor analysis studies look at the relationships between variables, as they are measured in individuals. This is R-technique and the columns of the correlation matrix are the variables in the study and the rows are the individuals used in the study. In any factor analysis, the columns of the matrix dictate the mode that is being factored and the rows contain the mode across which the factored mode is being measured. Although the modes that could be utilized in a study are numerous, you can never have the same mode as both rows and columns (Gorsuch, 1983). The six types of two-mode factor analysis are R-, Q-, O-, P-, S-, and T-techniques. Table 1 presents the six different kinds of two-mode factor analysis.

According to Gorsuch (1983), the different factor analysis techniques seek to identify different kinds of factors. By using variables as the factored items, P- and R-

techniques look for states and traits. O- and T-techniques, however, look for situations or environments, as the factored entities in these analyses are occasions. Q- and S-techniques factor subjects and try to identify different types of people. Choosing the method that is most appropriate depends on the research questions and the theory that is being followed in the research. As with any research, it is important to match the design of the study with reality.

Although the same raw data could theoretically be used in studies utilizing different techniques but the same two modes (e.g., variables and people), the data are organized differently depending on the technique and the reason for the research. Tables 2 and 3 display a heuristic demonstration of how the same data would be organized differently for two separate analyses. In the example, Q- and R-techniques are used. For the R-technique the columns of the data matrix are defined by the variables and the rows of the matrix are defined by the individuals in the study. For Q-technique the inverse is true. In this second model, the individuals in the study define the columns and the variables define the rows.

Although the two factor analysis procedures that are defined by the same two modes would yield the same results if simply inverted, it must be determined beforehand which factor analytic design is going to be utilized. This is necessary because there must be more rows than columns when using any factor analysis technique (Thompson, 1981). For example, when utilizing a Q-technique, there would need to be

many more variables than subjects or when using a P-technique there would need to be many more variables than occasions. A rule of thumb for this is that the number of columns should be no more than half of one less than the number of rows (Thompson, 1981). Therefore, if you had 41 individuals in an R-technique, you could only have 20 variables [(41-1)/2 = 20], or in a Q-technique you could have 95 variables but only 47 subjects [(95-1)/2 = 47].

There must be many more row entries than columns because this allows the relationships between the entities in the columns to be replicated many times over the rows. This builds evidence that the relationships within the factored mode are stable and therefore the factors that are extracted from the analysis are also somewhat stable (Thompson, in press).

Regardless of the technique used, the mode that is held constant determines the generalizability of the study. For example, in P-technique the two modes considered are occasions and variables, which assumes that the individuals mode is constant. While many designs will utilize a single case for a P-technique study, there are other ways that modes can be held constant, without actually using only one individual, occasion, or variable in the study. For example, the individuals mode could be held constant by "collapsing" across people, or finding one number to represent several people utilized in a study (e.g., the mean score for all individuals). According to Gorsuch (1983),

> ...modes of little interest are held constant, either implicitly or explicitly. The assumption made is that any generalization from the (R-technique) factor analysis will be only to the conditions under which the variables were collected for the initial factor analysis or to conditions sufficiently similar so that little loss of accuracy would occur. (p. 311)

Some researchers choose to utilize a three-mode technique for factor analysis, which takes into consideration all three modes, including variables, occasions, and individuals to counter the problem of limited generalizability. Three-mode factor analysis, however, is not used as widely as two-mode factor analysis, possibly because of the difficulty involved in interpreting the results of the analysis. Also, procedures for conducting a three-mode factor analysis are insufficiently developed at this time (Gorsuch, 1983). Perhaps most importantly, any of the six two-mode factor analytic methods (e.g., R-, Q-, S-) can be implemented with commonly available software, such as SPSS. However, specialized software that is not commonly available is necessary to implement the three-mode techniques.

Different Two-mode Factor Analysis Techniques

O- and P-technique

Factor analyses that consider occasions and variables, holding the individuals mode constant, are O- and P-techniques. Using occasions as the factored entity and variables as the replicates is an O-technique factor analysis. Jones, Thompson, and Miller (1980) offer a study utilizing an O design that evaluated how teachers perceive various models of teaching.

The inverse of O-technique is P, which also considers occasions and variables, holding individuals constant. However, P-technique factors variables across occasions. The use of P-technique is growing, now that many investigators are recognizing the utility of single-case or "N of 1" research designs (Gorsuch, 1983). Designs that study only one person while looking at many variables over several points in time constitute P-technique methodology. Examples of P-technique designs are studying a single individual's biorhythms based on multiple physical characteristics such as heart rate, blood pressure, and pulse over several occasions (Nunnally & Bernstein, 1994). Cattell (1953) also utilized a P-technique design in his research studying trends in Great Britain culture over the 100-year period between 1837 and 1937.

S- and T-technique

S- and T-techniques consider occasions and individuals, holding variables constant. Factoring occasions across

subjects is a T-technique factor analysis. The inverse of this is factoring subjects across occasions, which is S-technique. S and T-technique designs are rarely used in research. According to Gorsuch (1983),

> S and T techniques are virtually never used because the generalization would be limited to one variable. One suspects that the lack of S and T applications is a function of the fact that factor analysis comes, historically, from the individual difference approach to psychology and therefore experimentalists interested in only one characteristic have not been aware of its possibilities. (pp. 312-313)

Although S- and T- techniques are seldom used in research, Frankiewicz and Thompson (1979) offer a study of teacher brinkmanship behaviors that utilized the T design.

Q- and R-technique

R- and Q-techniques are the most commonly used factor analysis models. R-technique is the most used model and it studies relationships between variables, as they are measured across individuals. There are many reasons why R may be the most commonly used technique. For example, it is usually easier to get a large amount of people to do a small number of things and R is the easiest of the techniques to conceptualize (Nunnally & Bernstein, 1994).

Utilizing the same two modes, Q-technique factor analysis looks at how people group together, depending on their responses to certain variables. Q is useful for exploring data to find new types of people and for collecting data to confirm or disconfirm existing theories (Thompson, in press). Many researchers inappropriately use R-technique to identify types of people when a Q-technique would be more appropriate. According to Thompson (in press),

> Q-technique isolates types (or prototypes) of people. In fact, we often are more interested in types of people than we are in clusters of variables. For example, we often hear educators and psychologists talk about "Type A Personalities," "Workaholics," and "Introverts."

Examples of Q-methodology are Thompson and Miller's (1978) study of teacher education students' attitudes about different teacher types, Thompson's (1980) analysis of four types of evaluators, and Fielder's study of therapeutic relationships within the context of three different therapeutic approaches (1950).

Special Considerations for Q-technique

The mathematical computations for all types of two-mode factor analysis are identical. However, when using a Q-technique factor analysis there are some additional

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considerations for (a) subject selection, (b) stimuli selection, and (c) data collection.

After choosing Q-technique for the analysis, the researcher must answer three questions. First, it must be determined who should be factored (i.e., who the subjects in the study should be). Second, the researcher must decide which variables should be used to help define the person factors. Third, the researcher must consider which Q-technique should be used (Thompson, in press).

Subject Selection

When conducting a Q-technique factor analysis the researcher must carefully select the subjects for the study. Because the technique is based on the theory of identifying person types, the subjects must be chosen for the study depending on a known characteristic that they possess. As Kerlinger (1986) stated, "...one tests theories on small sets of individuals carefully chosen for their 'known' or presumed possession of some significant characteristics" (p. 598). Identifying people who possess certain characteristics often adds a second step to the analysis. Thompson (1980) utilized a two-step procedure in his analysis of an evaluator typology. The first step in the process involved categorizing the subjects of the study into four evaluator types based on judgments from their supervisors. Once the subjects were classified into evaluator types, they completed the second step in the procedure: the Q-sort task.

Determination of Stimuli

There are usually about 60 to 120 stimuli for the Q-technique (Kerlinger, 1986). There are various forms that the stimuli for the Q-technique can be in. For example, cards with writing or pictures on them, figures, or objects can be used as stimuli. These stimuli are arranged by the subjects in the study, which is called a Q-sort. The stimuli for the Q-sort can be selected in two ways, following a structured or unstructured model (Kerlinger, 1986). In a structured method the researcher uses sets of variables as stimuli that represent elements of a theory or hypothesis that is being tested. Thompson (1980) used a structured variable selection procedure when he chose the variables to study different evaluator typologies, as the stimuli were designed to test a particular theory. In contrast, an unstructured method for choosing variables would allow the researcher to use a large number of randomly chosen variables that are not directly testing a particular theory. In this method, the stimuli are assumed to measure one broad variable, for example, children's general attitudes about children with disabilities. This is the most common method for selecting variables for the Q-sort.

Three Models for Data Collection

When using a Q-technique the researcher must also determine how to conduct the data collection. The typical Q-sort asks individuals to put several items into categories

depending on some criterion (e.g., like most to like least). The subjects are told exactly how many items should be in each pile, which generally forms a normal or quasi-normal distribution. The quasi-normal distribution is good because the mean, SD, distribution is the same for all individuals. Kerlinger (1986, p. 509) offers the following sample distribution that could be used for a set of 90 stimuli in a Q-sort:

n items	3	+ 4	+ 7	+ 10	+ 13	+ 16	+ 13	+ 10	+ 7	+ 4	+ 3 = 90
Category	10	9	8	7	6	5	4	3	2	1	0

Normalizing the distribution corrects for the affect that shape can have on the relationship between the factored entities. Because the Pearson r correlation asks both if subjects ranked the stimuli in the same order and if there are equal gaps between the stimuli, the correlation is affected by the shape of the distribution of the stimuli. If the shapes of the distribution of stimuli for different subjects vary considerably, the Pearson r correlation coefficient is forced closer to zero and a perfect correlation cannot be obtained. By normalizing the distribution, the associations among people are not attenuated by the shapes of the distributions (Gorsuch, 1983).

A problem with the quasi-normal Q-sort is that while normalizing the distribution means the relationships between people is unaffected by the shapes of the distribution, the forced-choice response format takes away some of the variance

between individuals. For example, once cards are placed in a particular category, it is assumed that the individual doing the Q-sort feels exactly the same about all of the stimuli in the category, which is rarely true. This is of concern because more variance in scores can increase the reliability of the analysis. Therefore, some techniques have been proposed that put additional variance into the analysis by adding back information that is lost in the traditional Q-sort. The two additional methods for the Q-sort that have been proposed are the mediated Q-sort and the unnumbered graphic scale.

The mediated Q-sort is the same as classic Q-sort, but after the piles are formed the subjects then rank the items in each pile (Thompson, 1980). This procedure is attractive because it allows variance between scores to be added in after the original sorting has been completed. The two-step procedure is also helpful because it produces more accurate results than just having the subjects rank order a set of 60 to 120 stimuli, which would be time-consuming and frustrating.

The unnumbered graphic scale (Freyd, 1923) has subjects rate items on a scale anchored at both ends but with no numbers or descriptions between the anchors (Thompson & Dennings, 1993). The procedure asks subjects to rate the stimuli on a continuum. For example, under each item a four-inch line might be drawn between "DISAGREE" and "AGREE" and the subjects might then be told, "For each item, draw through

the line below each item at the point that best represents how much you agree or disagree with each item." In addition to adding variance to the procedure, a benefit of the unnumbered graphic scale is that there is independence between items (i.e., the response to one stimuli is not affected by the response to other stimuli in the set). However, as with the mediated Q-sort, the unnumbered graphic scale does not have the benefits making all distribution shapes the same and so score differences may be affected by the varying shapes of the different subjects' distributions.

Summary

In conclusion, there are many two-mode techniques for factor analysis that can be utilized in research. Several studies have utilized alternative two-mode factor analysis designs to identify types of people (Q- and S-techniques), states and traits (P- and R-techniques), and situation or occasion factors (O- and T- techniques). Although most studies use R-technique factor analysis, other models, such as the Q-technique, may be a more appropriate choice for many research designs. The computations for all two-mode factor analytic designs are identical, and therefore making an appropriate match between the research question and the research design does not require any extra effort by the researcher or special software. As with any research, it is important to match the research design with the reality that one is studying. Utilizing alternative forms of two-mode

factor analysis can assist with making this important match of research analysis to research questions.

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Table 1

Different Factor Analysis Techniques

Technique	Columns	Rows
R	Variables	Individuals
Q	Individuals	Variables
S	Individuals	Occasions
T	Occasions	Individuals
O	Occasions	Variables
P	Variables	Occasions

Table 2

Raw Data Matrix for a R-technique Factor Analysis

	V1	V2	V3	V4
Bob	1	2	3	4
Bruce	2	2	1	1
Brian	1	3	3	1
Brigitte	4	4	2	2

Table 3

Raw Data Matrix for a Q-technique Factor Analysis

	Bob	Bruce	Brian	Brigitte
V1	1	2	1	4
V2	2	2	3	4
V3	3	1	3	2
V4	4	1	1	2